OSCILLATORY INSTABILITY OF CONVECTIVE MOTION IN A HORIZONTAL LAYER OF GAS

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Results of an experimental investigation of oscillatory subcritical motion in horizontal layers of gas of different geometrical dimensions when there is loss of stability in steady roll-shaped flow are presented.

Theoretical and experimental investigations of the stability of steady convective motion in layers of liquid for different forms of perturbations have shown that two modes of instability exist: monotonic and oscillatory. The conditions for the oscillatory perturbations to develop were found in [1, 2] for natural convection in vertical layers. It was established that oscillatory disturbances lead to oscillatory convective motions. Investigation [1] revealed the important effect which the geometrical parameters of a vertical layer have on the type of developing disturbances.

In horizontal layers of liquid uniformly heated from below, for Prandtl numbers $Pr \geq 7$, the convective roll-shaped flow manifests itself initially as a monotonic instability [3, 4]. Monotonic disturbances cause a rearrangement of the stationary rolls in three-dimensional convective motion ($Ra \approx 12Ra_{cr}$), which become unstable to oscillatory disturbances when the Rayleigh number is increased. For a liquid with $Pr < 7$ the oscillatory instability of the convective motion occurs at lower Ra numbers ($Ra = 5600$-6000) due to loss of stability of the stationary rolls [4-7]. It was shown in [7] that the development of oscillatory disturbances occurs in the form of time-periodic bends of the rolls with respect to their horizontal axes. Investigations described in [3-7] were made in the layers the horizontal dimensions of which were greater than 20 times the height, i.e., the layer can be regarded as infinite in any horizontal direction. In this case no preferential orientation of the convective rolls, observed in narrow layers, was present. In a narrow rectangular band the preferred form of convective motion is two-dimensional rolls with axes parallel to the short side of the layer [8, 9]. An ordered structure of the convective motion can enhance its stability to oscillatory disturbances.

The present paper describes an experimental investigation of the stability of steady roll-like motion in horizontal layers of air of different geometrical dimensions.

The layer of air being investigated was bounded underneath by a brass plate with a Nichrome heater. The upper boundary of the layer consisted of a heat exchanger made up of two plates of Plexiglas in the gap between which there was a flow of water from a thermostat. The maximum horizontal dimensions of the layer $l \times d$ were 100×400 mm. To change the width of the layer d we used inserts of Plexiglas of known height. In constructing the gap we provided for the possibility of changing the height of the layer h over wide limits. The experiments, the results of which are presented in this paper, were carried out at layer heights h = 14.9, 20.5, 22.1, and 25.2 mm. The relative width of the layer d/h varied in the range 4.8-22.

The vertical temperature gradient was determined and the uniformity of the temperature distribution on the boundaries of the layer was checked using 32 copper–Constantan thermocouples. In all the experiments the divergence from the mean temperature of the horizontal layers did not exceed 2%. The temperature oscillations in the layer were recorded with a probe consisting of 7 copper–Constantan thermocouples (wire diameter 0.05 mm) with a time

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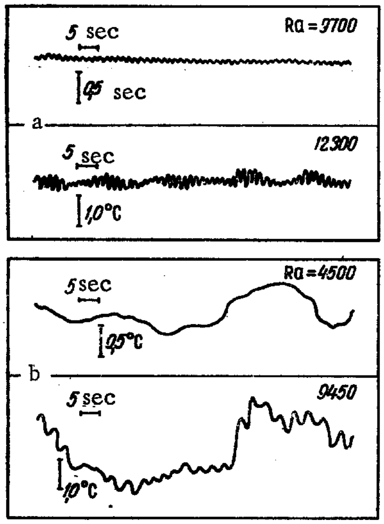

Fig. 1. Typical variations of the temperature with time at half the height of the layer (a, d/h = 5.7; b, 17.2).

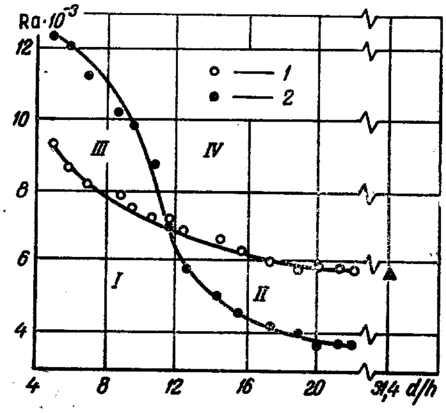

Fig. 2. Picture of the oscillatory instability of convective motion: 1) curve representing the instability to high-frequency disturbances; 2) curve representing the instability to low-frequency disturbances.

constant of about 0.2 sec, placed in the middle of the horizontal cross section of the gap. The signal from the thermocouples was amplified in an F359 photocompensation amplifier and recorded on the tape of a self-recording ÉPP-09 potentiometer. Visual observations of the convective motion were made through the upper transparent heat exchanger. Tobacco smoke was added to the layer of air for visualization purposes, the fine particles of which gave a contrast pattern when illuminated with a helium–neon laser.

The main error in calculating the Rayleigh criterion

$$\mathrm{Ra} = \frac{g\beta h^3}{\nu a}\Delta T$$

was due to the error in determining the height h and the nonuniformity of the temperature distribution on the horizontal boundaries. The deviations of the height from the mean value did not exceed ±0.1 mm. The maximum relative error in measuring ΔT was approximately 5%. The overall maximum relative error in calculating Ra was 8-9%. In all the calculations the physical properties of the air were determined at the mean temperature in the layer.

The possibility of local temperature nonuniformities arising on the horizontal boundaries of a material with low thermal conductivity (the thermal conductivity of Plexiglas λ_{Pl} = 0.184 $W\cdot m^{-1}\cdot deg^{-1}$) after a loss of static stability has been discussed in [3]. Such nonuniformities may have an inverse effect on the convective motion and stimulate a disturbance of the stability of the steady convective motion. In the present investigations we carried out a series of control experiments in which the upper heat exchanger was replaced by a brass plate (λ_b = 105 $W\cdot m^{-1}\cdot deg^{-1}$). The values of the Rayleigh criterion for which oscillatory instability was observed to occur agreed with the results obtained in similar experiments using a heat exchanger of Plexiglas. Hence, we did not detect any effect of possible nonuniformities of the temperature of the boundaries of Plexiglas on the oscillatory instability of the convective motion.

The method of determining the conditions under which oscillatory instability arises consisted in determining the temperature oscillations in the layer. The temperature mode in the layer was slowly changed by gradually increasing the voltage applied to the electric heater in the lower boundary. In this case the steady value of the Rayleigh number differed from the preceding one by 6-7%. An analysis of the continuous record of the temperature at half the height of the layer showed the existence of two types of oscillatory instability of the convective motion: high-frequency and low-frequency. The period of the high-frequency temperature oscillations in the layer lay in the range 1-10 sec. The low-frequency temperature oscillations were recorded with a period from tens of seconds to several minutes.

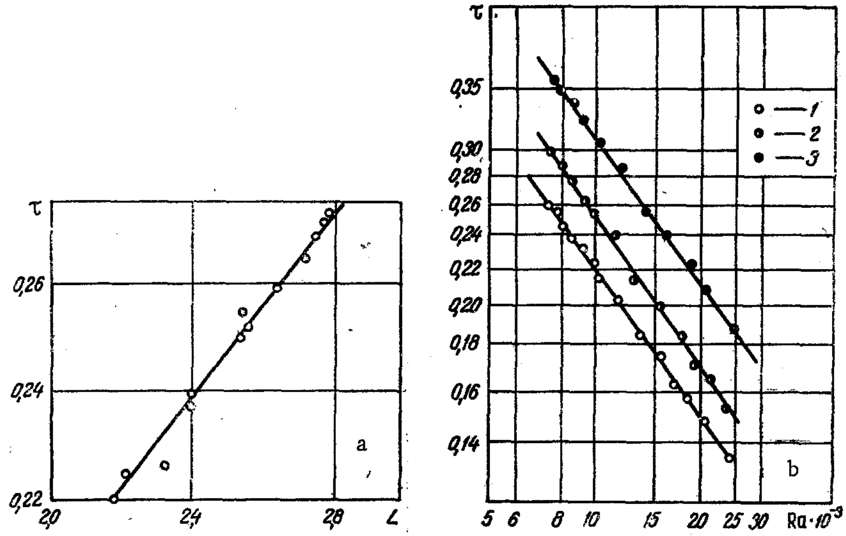

Fig. 3. Dependence of the period of the high-frequency oscillation in units of h^2/a on the spatial period of the bending of the rolls L, dimensionless with respect to h (a), and on the Rayleigh number Ra (b). For b) 1, d/h = 4.8; 2, 8.5; 3, 20.

The threshold of instability of the steady flow and the nature of the instability depended on the relative width of the layer d/h. In a narrow layer (d/h < 11) we first recorded high-frequency temperature oscillations (Fig. 1a). At higher Rayleigh numbers we detected the second type of oscillatory instability. The low-frequency temperature oscillations in this case amplitude-modulated the high-frequency oscillations. In a wide layer (Fig. 1b) for the same vertical temperature drops we observed the appearance of oscillations at low frequency, and then the high-frequency disturbances developed on the background of the low-frequency temperature oscillations. Consequently, the geometrical dimensions of the layer have an effect not only on the threshold of the oscillatory instability, but they also determine the different types of oscillatory instability of the steady roll-type flow.

As a result of experiments over a wide range of variation of the parameter d/h representing the geometrical dimensions of the layer we obtained a picture of the stability of the roll-type convective motion (Fig. 2). Curve 1 connecting the experimental points defines the boundary of the stability with respect to high-frequency oscillatory disturbances, while curve 2 is for low-frequency disturbances. Hence, in region I steady convective motion is observed, and in II there is low-frequency oscillatory instability. Region III corresponds to high-frequency oscillatory instability, while IV corresponds to joint development of oscillatory disturbances of both types. It is seen from Fig. 2 that when the width of the layer is reduced the stability of the convective motion increases. Whereas the stability of the convection rolls to high-frequency disturbances increases fairly smoothly when the parameter d/h is reduced, for the low-frequency disturbances for d/h ≈ 11 a sudden increase in stability occurs. Temperature oscillations at high frequency for d/h ≥ 19 were recorded for a Rayleigh number Ra = 5800, which is in good agreement with the results obtained in [7] (the triangle on the stability pattern). Our experiments showed that the value of the parameter d/h ≥ 19 corresponds to conditions when the horizontal layer has no boundaries when investigating oscillatory instability of convective motion: curves 1 and 2 of Fig. 2 for d/h ≥ 19 become parallel to the abscissa axis.

By means of visual observations we studied the mechanism of the loss of stability of steady flow. It was established that an increase in the Rayleigh number in a narrow layer (d/h < 11) to Ra values 10-12% lower than the threshold value leads to spatial bending of the convection rolls with respect to their axes. The amplitude of the bending increases when Ra is increased further. The threshold value of Ra for high-frequency oscillations corresponds to the beginning of displacement of the crests and troughs of the bent rolls. This process recalls the propagation of transverse waves in a stretched cord.

The following relation for the period of the high-frequency temperature oscillations has been established experimentally [7]:

$$\tau = 9.6 \cdot Ra^{-0.4} Pr^{-0.4}, \quad (1)$$

where τ is dimensionless with respect to h^2/a. The experiments carried out for different Prandtl numbers for liquids [3, 4] showed that the period of the high-frequency oscillations τ is proportional to $Ra^{-0.64}$. In a theoretical investigation [10] for a layer with free boundaries the following expression was obtained:

$$\tau \sim L\, Ra^{-0.5},$$

where L is the spatial period of the bending of the roll along the axis in units of h.

In the present paper we also measured the period of the temperature oscillations at high frequencies τ and the spatial period of the bending of a roll L. The investigation for a constant Rayleigh number (Fig. 3a) confirms the theoretical conclusion [10] regarding the linear relation between τ and L. The dependence of the period of the high-frequency temperature oscillations on the Rayleigh number for values of d/h of 4.8, 8.5, and 20 is shown in Fig. 3b. The results of the experiments can be approximated by the relation

$$\tau = 3.9L \cdot Ra^{-0.41}. \quad (2)$$

It was noted in [7] that the observed mean value of the spatial period of the bending of the rolls is L = 2.5 h for experiments on air. If we use relation (2), taking into account the dependence of τ on h, then we obtain from Eq. (1) for air (Pr = 0.707) a value L = 2.82 in units of h. Hence, within the limits of experimental error we can generalize Eqs. (1) and (2) to the form

$$\tau = 3.9L \cdot Ra^{-0.4} \cdot Pr^{-0.4}.$$

Visual observations showed that low-frequency oscillatory instability is realized in the form of a displacement of the convection rolls in the direction perpendicular to their axes. When d/h is increased the amplitude of the periodic displacement of the rolls increases for a fixed temperature mode of the layer. In all the experiments, when the width of the layer d was greater than 11 times the height h, we observed an equality of the double amplitude of the low-frequency temperature oscillations of the temperature difference in the updraft and downdraft of the convective flows.

NOTATION

l, length of the layer; d, width of the layer; h, height of the layer; g, acceleration due to gravity; β, coefficient of thermal expansion; ν, kinematic viscosity; a, thermal diffusivity; λ, thermal conductivity; ΔT, temperature drop between the horizontal boundaries of the layer; τ, period of the high-frequency temperature oscillations; L, spatial period of the bending of the rolls; Ra, Rayleigh number; Pr, Prandtl number.

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